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SUMMARY

An analysis of steady-state pressure-drop characteristics in long, slender, circular tubes related to the "laminar-instability" problem is presented. Results obtained by detailed numerical computations are based on real-fluid para-hydrogen properties and assumed laminar to turbulent and turbulent to laminar flow transition criteria.

Correlation parameters are derived from approximate solutions of flow and heat-transfer relations and are used to generalize calculated pressure drop over the Reynolds number range of mixed turbulent and laminar flows.

Results indicate that the minimum pressure drop occurs within or near the range of mixed turbulent and laminar flows. For the long, slender tubes considered, therefore, a rule of thumb is suggested: The minimum pressure drop, and consequently the potential instability, occurs just below the flow where tube exit conditions become laminar. The temperature ratio of the fluid at the point of minimum pressure drop varies upwards from the values of about 5, predicted by laminar flow criteria, to 15, for example. As a result, heat exchangers may not encounter laminar-instability problems until temperature ratios are higher than previously expected.

The range of conditions investigated includes inlet pressures of 1 to 1000 pounds per square inch absolute, inlet temperatures of 50° to 150° R, tube lengths up to 6 feet, and tube diameters from 0.05 to 0.3 inch. Heat input is varied to produce temperature ratios up to 80, and inlet flow or Mach number varies over a range of four decades.

It is shown that the laminar instability constitutes little problem at high pressure levels (hundreds of lb/sq in.) where design Mach numbers should exceed the values at minimum pressure drop by factors of hundreds. At low pressures, for example, below atmospheric pressure, the minimum pressure drops and the associated inlet Mach numbers occur throughout the range considered practical operating conditions. Consequently, the laminar instability is potentially present during steady operation at low pressures whenever the wall- to fluid-inlet temperature ratio is high.

INTRODUCTION

The application of nuclear energy to rocket propulsion introduces the use of high-power solid to gas heat exchangers with large temperature ratios, for example, 40, and with thousands of parallel flow passages. The tendency of such systems to amplify maldistributions in flow and temperature conditions is generally recognized. The inherent sensitivity of high-temperature-ratio heat exchangers reflects directly on the average propellant temperature obtainable, within given material constraints, to limit the performance of the propulsion system.

One facet of the overall design problem is the instability of laminar flow in parallel passages, or the so-called "laminar-instability" problem. Various analyses of this problem are available in the literature, for example, references 1 and 2. Because the difficulty in maintaining favorable flow and temperature distributions with laminar flow is well known, it is normal to design heat exchangers to operate with turbulent flow. Nevertheless, laminar flow will most certainly be encountered in the heat exchanger during low-flow operation, either for low-power continuous running or as is associated with startup or shutdown cycles.

In the published analyses, such as those of Gruber and Hyman (ref. 1) and Bussard (ref. 2), it is conventional to associate the potential laminar instability with negative values of the rate of change in pressure drop with respect to weight flow at constant heat input; that is, with assumed steady flow and constant heat addition, <u>decreases</u> in the mass flow yield <u>increases</u> in the pressure drop. Although the <u>dynamic</u> behavior under these conditions is not clearly defined, it is generally agreed that the instability phenomenon is related to the negative slope. Gruber and Hyman show that the negative slope occurs only with laminar flow and temperature ratios exceeding 3.7 for air (or 4.7 for hydrogen).

This report presents a study of the pressure-drop characteristics of interest and employs three techniques of analysis:

(1) Steady-flow and heat-transfer relations are solved on the basis of real-fluid para-hydrogen properties by numerical integration (aided by a digital computer program). Particular emphasis is given to the transition from turbulent to laminar flow along the tube that results from increased viscosity at higher temperatures. The calculations are iterative (trial and error) and use an <u>open</u> formulation.

(2) An approximate solution is obtained in <u>closed</u>, integrated form that is similar to those of references 1 and 2. Trends based on laminar flow are presented and discussed; however, laminar flow conditions often do not occur in the range of interest, and therefore the trends do not generally apply.

(3) The results of the closed-form solution are used to derive correlating parameters, which are then used to correlate the results of the numerical calculations for laminar, mixed, and turbulent flow conditions.

The reader is cautioned to use care in drawing conclusions based on the correlation of analytical results without verification from experimental data.

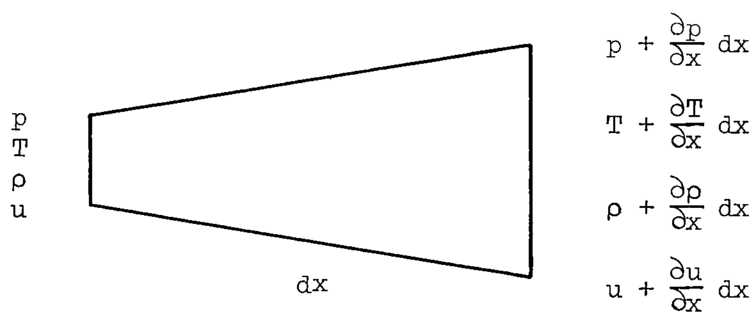
Tubes ranging from 0.05 to 0.3 inch in diameter and from 2 to 6 feet in length are considered for pressure levels from 1 to 1000 pounds per square inch absolute and inlet temperatures from 50° to 150° R. Power inputs are varied consistent with para-hydrogen temperature ratios up to 80.

ANALYSIS

Consider first the steady flow of a homogeneous fluid in a segment of heat-exchanger passage that is essentially one dimensional; that is, the y- and z-components of velocity u are negligible. (All symbols are defined in appendix A.) For a fluid density ρ and a cross-sectional area A the conservation of mass is expressed as

$$\frac{d}{dx} \rho A u = 0 \tag{1}$$

and the weight flow rate w is constant. The conservation of momentum in the



absence of mechanical or gravitational forces is expressed as

$$\frac{1}{\rho}\frac{dP}{dx} + \frac{u}{g}\frac{du}{dx} + \frac{f}{r_h}\frac{u^2}{2g} = 0 \tag{2}$$

where f is the Fanning friction factor and the hydraulic radius r_h is 1/4 diameter for a circular cross section. It is convenient to refer the heat input to both the weight of fluid and the heat-transfer surface area. The terms are related as

$$Q = \rho q r_h \tag{3}$$

where

Q power per unit of surface area, Btu/(sec)(sq ft)

q power per pound of fluid, ft/sec

The entropy change in the fluid is related to the heat input q and the entropy increase due to friction by

$$\frac{q}{u} + \frac{f}{r_h}\frac{u^2}{2g} = T\frac{ds}{dx} \tag{4}$$

or, substituting for the entropy change (ref. 3, e.g.) gives

$$\begin{aligned}\frac{q}{u} + \frac{f}{r_h}\frac{u^2}{2g} &= c_v\frac{dT}{dx} + T\left(\frac{\partial p}{\partial T}\right)_v\frac{dv}{dx} \\ &= c_p\frac{dT}{dx} - T\left(\frac{\partial v}{\partial T}\right)_p\frac{dp}{dx} \\ &= \frac{dH}{dx} - \frac{1}{\rho}\frac{dp}{dx}\end{aligned} \tag{5}$$

These relations, which represent the conservation of mass (eq. (1)), momentum (eq. (2)), and an energy relation (eq. (4)), form the basis of both the closed-form and numerical results.

NUMERICAL INTEGRATION

Numerical integration of the flow equations is performed by stepwise iteration techniques. A state equation is solved independently at each iteration step (in open form) as

$$p_i = p_i(\rho_i, T_i)$$

The values of p, c_v, H, μ, and k are approximately the para-hydrogen properties discussed in references 4 to 7 and are computed as shown in reference 8.

The Fanning friction factor f for the computations herein is assumed as follows:

Laminar flow:

The Poiseuille form is

$$f = \frac{16}{Re} \tag{6a}$$

Turbulent flow:

The Kármán or Kármán-Nikuradse formulation with modifications for use with high film- to bulk-temperature ratios ($T_f/T_b > 1$) as suggested in reference 9 is

$$\frac{1}{\sqrt{f\,\frac{T_f}{T_b}}} = 4\,\log\left(Re\,\frac{\rho_f}{\rho_b}\sqrt{f\,\frac{T_f}{T_b}}\right) - 0.4 \tag{6b}$$

In both cases, the Reynolds number is evaluated as

$$Re = \frac{D\rho u}{\mu_f} = \frac{4r_h\rho u}{\mu_f} = \frac{4r_h}{\mu_f}\left(\frac{w}{A}\right) \tag{7}$$

where the film viscosity μ_f is evaluated at $T_f = (T_w + T_b)/2$, the average of wall and bulk temperatures. Flow is considered laminar at Reynolds numbers up to 2100 or turbulent down to about 1000 with the hysteresis loop shown in figure 1. Therefore, a laminar flow remains laminar until the Reynolds number exceeds 2100, and turbulent flow remains turbulent until the friction factor in laminar flow is larger than that in turbulent flow.

For steady flow, with the implicit assumption of <u>no</u> wall heat capacity, the power transferred to the fluid is directly the power generated in the solid heat exchanger, or

$$q_{generated} = q_{to\ fluid}$$

The wall temperature is then computed from

$$Q = h(T_w - T_b) \tag{8}$$

The surface coefficient h is assumed as follows:

Laminar flow (ref. 10):

$$\frac{hD}{k} = 1.75\left(\frac{wc_p}{kx}\right)^{1/3} \tag{9a}$$

where x is the distance from the entrance to the passage.

Turbulent flow (ref. 11 and others):

$$\frac{hD}{k} = 0.023\ Re^{0.8}Pr^{0.333}\left[1 + \left(\frac{4r_h}{x}\right)^{0.7}\right] \tag{9b}$$

The terms are evaluated at film conditions, and the hysteresis in laminar-turbulent transition is as stated for the friction factor. The length x in equations (9a) and (9b) is constrained to equal or exceed 1 hydraulic radius.

Closed-Form Approximation

An approximate solution of the flow equations is shown in appendix B. The following assumptions are introduced in the derivation:

(1) The fluid is a perfect gas, $P = \rho RT$.

(2) The Mach number is low, $M^2 \ll 1$.

(3) The friction factor f is approximated as $f = f_0/Re^n$.

(4) Viscosity μ is an exponential function of temperature as $\mu = \mu_0 T^m$.

(5) The heat input distribution is constant along the length of the passage, so that $Q(x) = Q_0$.

The integrated relation for pressure drop is, from appendix B,

$$\frac{\Delta p}{p} = \gamma M^2 \left[\tau - 1 + \frac{f_1 x}{2r_h} \frac{\tau^{mn+2} - 1}{(mn + 2)(\tau - 1)}\right] \tag{10}$$

where the fluid temperature ratio τ is

$$\tau = \frac{T_2}{T_1} \tag{11}$$

The first term represents the momentum pressure drop and the second the frictional pressure drop. Because both terms include the coefficient M^2, a term previously assumed small, the region of validity is implicitly a region of small pressure drops.

The algebra of differentiating the relation for $\left(\frac{\partial\, \Delta p/p}{\partial\, w/A}\right)_{Q,p_1,T_1}$ is given in appendix B. It is shown that

(1) The momentum pressure drop always increases with weight flow but is generally small with respect to the frictional pressure drop.

(2) The frictional pressure drop can decrease with increases in weight flow whenever the temperature ratio is large and $n(m + 1) > 1$, where m and n are the exponents describing the variation of viscosity with temperature and f with Re, respectively.

(3) The negative slope in frictional drop will occur for condition (2) whenever

$$\tau > -\frac{mn + 2}{1 - mn - n}$$

These stability criteria for the frictional pressure drop are plotted for a series of temperature ratios in figure 2(a). Below the curve for $mn + n = 1$, the flow would be stable. The flow tends to become unstable (1) as the viscosity becomes a stronger function of temperature; that is, m increases, (2) as the friction factor becomes a stronger function of Reynolds number (n increases), and (3) as the temperature ratio increases.

For most gases, the viscosity varies with temperature to powers less than 1. For hydrogen, m is in the range 1/3 to 1. Turbulent flow with Reynolds number exponents less than 1/2 is always stable.

Because the stability of flow in the laminar flow regions, n = 1, is dependent on the temperature ratio τ, the stability criteria are shown in greater detail in figure 2(b). Again, increases in viscosity variation with temperature or temperature ratio tend to make the flow unstable. With respect to frictional pressure drop, if the momentum pressure-drop terms are ignored, as shown in figure 2(b), the flow could become unstable at temperature ratios down to 3 for an m of 1. Also, for temperature ratios greater than 7, the flow would most certainly be unstable for hydrogen gas in laminar heat exchangers.

The foregoing results have been obtained without the specification of length or hydraulic radius, and flow Mach number is restricted only as small with respect to 1. Thus, for arbitrary fluid properties, the potential instability can be avoided only by maintaining turbulent flow or by reducing the temperature ratio.

Correlation Parameters

The approximate pressure-drop relation (eq. (10)) forms the basis for developing parameters to be used in correlating pressure drop for long, slender, heated tubes. With the momentum pressure-drop term omitted, the frictional pressure drop is represented as

$$\frac{\Delta p}{p} = \gamma M^2 \frac{f_1 x}{2r_h} \frac{\tau^{mn+2} - 1}{(mn + 2)(\tau - 1)} \tag{12}$$

With various substitutions from appendix B, including perfect-gas relations, equation (12) is expressed as

$$\frac{D}{x}\left(\frac{Dgp}{C\mu}\right)^2 \frac{\Delta p}{p} = \frac{2f_0}{\gamma} \mathrm{Re}^{2-n} \frac{\tau^{mn}(\tau^{mn+2} - 1)}{(mn + 2)(\tau - 1)} \tag{13}$$

Resorting to functional variations and substituting for the temperature ratio from equation (B7a) yield

$$\frac{D}{x}\left(\frac{gDp}{C\mu}\right)^2 \frac{\Delta p}{p} = f\left(\mathrm{Re}, \frac{Qx}{\mu c_p T_1}\right) \tag{14}$$

Finally, assuming that the coefficients are evaluated at bulk fluid conditions at the tube inlet and introducing shorthand symbols for the groupings give

$$
\left.
\begin{aligned}
\psi &\equiv \frac{D}{x}\left(\frac{gDp_1}{C_1\mu_1}\right)^2 \frac{\Delta p}{p_1} \\
Re_1 &\equiv \frac{D\rho_1 u_1}{\mu_1} \\
\varphi &\equiv \frac{Qx}{\mu_1 C_{p,1} T_1} \\
\psi &= \psi(Re_1,\ \varphi)
\end{aligned}
\right\}
\tag{15}
$$

The general applicability of the correlation technique will be shown by using the results of numerical calculations.

RESULTS

In the following discussion, the validity of the correlation technique will be illustrated by attempting to generalize the results of open-form numerical computations with the parameters ψ and φ. Then, with the use of numerical methods alone, some additional aspects of the so-called laminar-instability problem will be considered. It is of particular interest here to evaluate the influence that real-fluid properties have on pressure drop and to inspect some possibilities related to the transition from turbulent to laminar flow. In the course of the discussion, a cursory examination will be made of the effects of tube diameter and length, pressure level, heat input, and inlet temperature.

Typical Instability Characteristic

To define the so-called laminar-instability problem further, a typical example of the pressure-drop characteristic obtained from numerical integrations is shown in figure 3. A specific set of conditions is assumed:

Circular-flow-passage diameter, D, in.	1/10
Length, x, ft	4
Heat input, Q, Btu/(sec)(sq ft)	1
Inlet temperature of hydrogen gas, T_1, $^{\circ}R$	50
Inlet pressure, p_1, lb/sq in. abs	20

Results are shown in terms of the drop in static pressure $\Delta p/p$ and of the inlet Mach number M_1 with logarithmic scales. The calculations do not include the effects of dissociation of the hydrogen molecules; the results are shown as dashed curves at temperatures where 5 percent or more error in specific heat is anticipated. For a nominal pressure level of 20 pounds per square inch absolute, dissociation becomes significant at about 3500° R, or at a temperature ratio of about 70, as shown in figure 3.

Results, however, do include the effects of flow-transition criteria, as indicated in figure 1. In figure 3 laminar friction factors and heat-transfer coefficients are used for inlet Mach numbers less than 0.0031 or 3.1×10^{-3}. At an inlet Mach number of 0.0082 and above, the flow is turbulent. In the transition region ($0.003 < M < 0.008$), flow is turbulent at the inlet as a result of assuming fully developed flow and diameter-based Reynolds numbers and transitions (changes) to laminar flow due to the temperature and the viscosity increase along the length of the flow passage.

Some insight into the significance of this problem is gained by using linear pressure-drop, temperature-ratio, and Mach number scales in figure 4 in contrast to the logarithmic scales of figure 3 for the same results. From the linear form, it is obvious that the positive slope associated with turbulent flow occurs over most of the Mach number range, and that the minimum pressure drop occurs at a very low flow rate. In addition, the value of the pressure drop at the minimum is low, namely, 1.2 percent.

Speculation on the nature of the laminar instability is possible if only steady-state computations are used. To this end, figure 5 shows the previous results and a curve representing equilibrium pressure drop at constant wall-temperature profile. (In figs. 3 and 4 the heat input Q was constant and the wall temperature T_w varied; in fig. 5 the curve with a fixed T_w profile has varying heat input.) When evaluated at a fixed wall-temperature profile, the pressure-drop curve has a positive slope and does not reflect potential instability. Therefore, the reversal in the constant heat input curve reflects a low-frequency instability and not an instability of higher gas-dynamic frequencies. In other words, for a disturbance too fast to allow the temperature of the wall to change, the constant wall temperature characteristic applies, and the slope is positive. Therefore, a stable condition exists.

It is reasonable to conjecture further that the so-called laminar instability is related to the response rates of the wall heat capacity. There is, in addition, experimental evidence indicating that the unstable condition is not cyclic in nature but is more characteristic of a "flow stoppage" (ref. 12). While this portion of the discussion is of interest in defining the problem, it is beyond the scope of the present analysis.

Evaluation of correlation parameters. - The effectiveness of the heat-input and pressure-drop correlating parameters in generalizing pressure-drop results over a range of inlet conditions, tube geometries, and heat inputs is shown in figure 6 for nominal values of the heat-input parameter φ of 3400 and 34,000. The pressure-drop parameter ψ is shown as a function of inlet Reynolds number on logarithmic scales. The mixed flow varies over a range of inlet Reynolds number from 2000 to 10,000 with the cases used.

Figure 6(a) illustrates primarily the correlation of results with varying inlet pressure. The results for a range of inlet pressure p_1 from 2 to 20 pounds per square inch absolute are correlated to about ±10 percent. All points for an inlet pressure of 1 and some for 2 pounds per square inch absolute (parametric variation, fig. 10) fell above the other points as much as 70 percent because of high exit Mach numbers and are not shown.

Figure 6(a) shows results for tubes 4 feet long and for a tube 2 feet long with twice the heat input Q to maintain the constant heat-input parameter φ. The points for tubes 2 feet long correlate well, but similar points (not shown) for 1-foot-long tubes deviate significantly.

The correlation of pressure-drop results obtained by numerical computation is shown in figure 6(b) for various configurations having a heat-input parameter of 34,000 ±2 percent.

For the variations in figure 6(b), the results generalize to about ±30 percent. The range in pressure drop correlated at each Reynolds number was about 100 to 1 as a result of the assumed parametric variations.

Briefly, heat-input-parameter, Reynolds number, and pressure-drop-coefficient correlations are valid for long, slender tubes with flow Mach numbers less than 0.3 and pressure drops less than 10 percent. The fluid must be nearly a perfect gas, and the heat-input profile along the length of the tubes should reasonably approximate a flat distribution.

It should be noted that the correlation parameters do not generalize changes in pressure drop due to changes in power distribution along the tube. In other words, results from similar distributions will correlate each other, but results from altered distributions may differ, depending on the size of the changes.

<u>Prediction of laminar instability</u>. - From the results shown it is possible to draw several conclusions related to the laminar instability problem:

(1) The conditions at minimum pressure drop are not well defined for two reasons. First, the pressure-drop curve (figs. 3 to 6) has a flat minimum that is difficult to define. Second, the particular values used to establish laminar-turbulent transition criteria are arbitrarily assumed.

(2) The minimum pressure drop occurs at flows and pressure drops just less than those at which the flow in the tube is entirely turbulent. A rule of thumb is suggested: <u>The laminar instability is potentially a problem whenever any part of the heated section has laminar flow.</u>

(3) The validity of predictions of temperature ratio at minimum pressure drop is also compromised by mixed flow and the flat minimum (as is (1) preceding). The temperature ratio at the minimum pressure drop, however, is often much higher than about 5, as is predicted on the basis of purely laminar flow. For example, the temperature ratio in figure 3 at the minimum pressure drop is about 17.

Influence of Design and Operating Conditions

Most of the principal conclusions arrived at as a result of this analysis have been indicated. It is of interest now to investigate by means of parametric variations whether these conclusions are general to a broad family of conditions and geometries, which may be of interest in nuclear-rocket heat-exchanger design. In all the following discussions, the working fluid is para-hydrogen, and the heat-input distribution to round tubes is constant in the axial direction.

Effect of heated length. - The effect of the length of the heat-exchanger tube is shown in figure 7 where the same format is used as was used previously. Where the sample is 4 feet long, integral lengths from 2 to 6 feet long are considered. As expected, pressure drop increases with length; the temperature ratio also increases with length. The transition from laminar to turbulent flow at the inlet is independent of length, but the transition to turbulent flow at the exit occurs at higher flows and longer lengths.

The significant result of figure 7 is that the length has little effect on the relation between minimum pressure drop and the pressure drop at the transition to all turbulent flow.

Effect of tube diameter. - The influence of the assumed passage diameter D on the pressure drop is shown in figure 8. The range of D illustrated is from 0.05 to 0.30 inch, whereas the previous results considered only 0.10 inch. The tube length is again the nominal value of 4 feet.

For an equal heat flux per unit surface area, pressure drop and temperature ratio vary inversely with diameter, as expected. The region of mixed flow occurs at higher Mach numbers with small tubes because of the diameter in Reynolds number. Thus, to remove a given power from the tube surface, a change in tube diameter from 0.30 to 0.05 inch or a ratio of 6 will necessitate increasing the minimum Mach number (or weight flow/area, w/A) by about 6 times to avoid potential instabilities; however, the pressure drop will increase about 150 times. These changes occur at about constant outlet temperature. Note that the minimum pressure drop is again in the range of the transition to all-turbulent flow.

Effect of heat input at gaseous inlet conditions. - The effect of varying the level of the flat heat-input distribution is considered in two parts: (1) at low pressures where the inlet conditions are gaseous, and (2) at higher pressures where liquid hydrogen occurs at inlet conditions. Vapor-phase results are avoided herein by the choice of inlet conditions.

For gaseous hydrogen at 20 pounds per square inch absolute and for 50° R inlet conditions, the heat input is varied from 0.1 to 5 Btu per second per square foot, as illustrated in figure 9(a) in terms of pressure drop and temperature ratio, as shown previously. As anticipated, increasing the heat input to the fluid increases the temperature ratio, the pressure drop, and the Mach number at which the outlet conditions will remain laminar. The transition is at higher Mach numbers with higher heat input due to the higher temperature ratios and subsequently to higher film viscosity.

To prevent the tube outlet conditions from becoming laminar as heat input is increased 50 times as shown, the inlet Mach number must be increased about 4 times, the pressure drop must be increased about 100 times, while the outlet temperature will be increased from 200° to 1800° R for a change in temperature ratio of 12.

The approximation that the minimum pressure drop and the right boundary of the transition region are close together is noteworthy at high heat-input levels but less significant at low heat input. For example, with 0.1 Btu per second per square foot, the minimum occurs at a Mach number of about 0.002 relative to 0.0036 for the transition, and the pressure drop is about 10 percent less.

Effect of heat input at liquid inlet conditions. - Trends similar to those just presented are indicated in figure 9(b) for an inlet pressure of 500 pounds per square inch absolute, an inlet temperature again of 50° R, and a heat input from 1 to 100 Btu per second per square foot.

Of significance is the combination of high temperatures, low inlet Mach numbers, and low values of pressure drop associated with the transition to mixed flow. A discussion of the pressure effect is forthcoming but is illustrated by the lowest curve in figure 9(b). The higher curves are shown here to indicate the unlikely probability of encountering the laminar phenomena at a high pressure, high heat-input condition without prior overtemperature of the system.

Effect of pressure level at gas inlet conditions. - The approximate pressure-drop relation (eq. (10)) indicates that pressure drop should vary inversely as the outlet pressure. The anticipated trend is shown in figure 10(a) for pressures of 1, 2, 5, 10, and 20 pounds per square inch absolute and a heat input of 0.1 Btu per second per square foot.

The unusually low heat input is used to allow pressures as low as 1 pound per square inch absolute, since any higher heat input cannot be removed by the fluid without choking the flow at either the left end because of the temperature rise or at the right end of the Mach number curve because of the frictional pressure drop. For the low heat input used, the minimum pressure drop occurs at lower temperature ratios and possibly in the laminar flow range.

In relating the laminar-instability problem to a multiple-passage heat exchanger such as a reactor core, the "average" exit Mach number of the passages is related to the exit nozzle throat area and consequently to the "average" pressure drop imposed on any single tube under given conditions. Only as flow conditions leading to the instability occur at pressure drops in the range of average conditions, for instance, of the order of 10 percent, will the instability be likely to occur. These conditions are found at low pressure levels with very little heat input and at slightly higher pressures with higher heat input (e.g., fig. 9(a)).

Effect of pressure level at liquid inlet conditions. - Previous discussion has indicated that at high pressures the minimum pressure drop should be small and should occur at very low inlet Mach numbers. This indication is verified in figure 10(b) for pressures of 300, 500, and 1000 pounds per square inch absolute, where the minimum pressure drop is 1/10 percent or less and the accompanying Mach number is 0.00017 or less.

It is interesting to note, however, that the temperature-ratio curves for the three pressure levels tend to merge, whereas they normally would be expected to separate because of varying weight flow as pressure varies (e.g., fig. 10(a)). The nonlinearity is the result of expanding liquid into gas at supercritical pressures, where the term $T\left(\frac{\partial p}{\partial T}\right)_v \frac{dv}{dx}$ (eq. (5)) absorbs a much larger portion of the power input than with a gas to gas expansion. The rapid expansion takes place in the temperature range up to 100° to 200° R, and therefore the curves become parallel above temperature ratios of about 3, as expected.

Effect of inlet temperature at gas inlet conditions. - The influence of the inlet temperature of the para-hydrogen gas on the stability criteria is presented in figure 11(a). The region of mixed flow occurs at higher inlet Mach number with higher inlet temperature because of the increase in viscosity. Also, since the inlet temperature increase is larger than proportionate to the outlet temperature rise, the inlet (left) transition occurs at relatively higher Mach number, and the mixed-flow region decreases in size. The minimum pressure drop again occurs within the transition, however.

Effects of inlet temperature at liquid inlet conditions. - The pressure-drop and temperature-ratio curves for an inlet pressure of 500 pounds per square inch absolute are shown in figure 11(b). The effect illustrated is that of increasing the inlet temperature from liquid conditions at 50° R to compressed fluid conditions at 150° R. As anticipated, the pressure drops remain small; the minimum occurs at about 2×10^{-3} percent, or two parts in 10,000. As with the low pressures, increasing inlet temperature increases the Mach number at which the minimum pressure drop and the mixed-flow region occur.

The influence of the nonlinearities associated with the expansion of liquid into gas at supercritical pressures, as discussed in regard to the pressure effect in figure 10, is again evident in figure 11, particularly with respect to the temperature-ratio curves, which coincidentally reflect the compensating effects of changes in several variables.

CONCLUDING REMARKS

A steady-state one-dimensional analysis of the laminar-instability problem is described in relation to geometries typical of nuclear-rocket heat exchangers. Results are obtained by detailed numerical integration of the flow of real-fluid para-hydrogen in long tubes of small circular cross section. Criteria fixing the transitions between laminar and turbulent flow are assumed and, as used, lead to a hysteresis loop.

A closed-form approximate solution of the heat-transfer and flow relations is also presented. Correlation parameters are derived from the functional relations and are used to generalize the computed numerical results with reasonable accuracy, approximately ±30 percent. The correlating technique could, in addition, serve as a method of estimating pressure drops in long, slender tubes for flowing gaseous para-hydrogen at low Mach numbers.

Results of parametric investigations indicate that the minimum pressure drop always occurs within, or in the proximity of, the transition region; that is, where the inlet Reynolds number exceeds 2100 and the exit Reynolds number is less than 1000. Thus, the values at minimum pressure drop predicted by published analyses based on laminar flow are not applicable for the range of long, slender tubes studied. For example, minimum pressure drop may occur at a temperature ratio of 15 with mixed flow rather than about 5 with laminar flow. Other assumed transition criteria (within credible limits) will modify this result only in detail. Consequently, the following rule of thumb is suggested:

The minimum pressure drop, and consequently the potential instability, occurs just below the flow and pressure drop at which the exit of the passage becomes laminar.

Several design considerations can be reiterated:

1. The tube pressure drop, including the minimum pressure drop, varies inversely as the inlet pressure squared, and the minimum pressure drop occurs at higher Mach numbers with lower pressures.

2. The minimum pressure drop and the Mach number at which it occurs increase with heat input.

3. The tube length has relatively less effect; however, pressure drop and the Mach number at the minimum increase with length.

4. For a given heat input per unit surface area, the minimum pressure drop and the associated Mach number increase as the diameter is decreased.

5. It is suggested that the "instability" is not related to high-frequency gas dynamics and, thus, by inference is related to the wall capacity and heat input. Limited experimental evidence with single tubes indicates that a "flow stoppage" phenomenon, which is a relatively slow but irreversible process, results.

When the preceding conclusions are related to a design problem, it is first clear that during high-pressure operation at normal Mach numbers (> 0.01) the possibility of encountering laminar instabilities is slim indeed. During low-pressure operation, such as is anticipated for reactor aftercooling and potentially may be encountered during startup or shutdown procedures, the laminar-instability problem may represent a serious threat to structural integrity. Within the constraint of a particular design, the problem is most likely at low flows, high heat input, and subsequently at the high fluid-temperature ratios consistent with high wall- to inlet-fluid temperature ratios.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, September 6, 1963

APPENDIX A

SYMBOLS

A area, sq ft

C sonic velocity, ft/sec

c_p specific heat at constant pressure, Btu/(lb)($^{\circ}$R)

c_v specific heat at constant volume, Btu/(lb)($^{\circ}$R)

D diameter, ft

f Fanning friction factor

g standard acceleration due to gravity

H enthalpy, Btu/lb

h surface coefficient in convection, Btu/(sec)($^{\circ}$R)

k thermal conductivity, Btu/(ft)(sec)($^{\circ}$R)

M Mach number

m exponent of viscosity variation with temperature, eq. (B6)

n exponent of friction factor variation with Reynolds number, eq. (B5)

p pressure, lb/sq ft

Pr Prandtl number

Q heat input per unit surface area, Btu/(sec)(sq ft)

q heat input per pound of fluid, eq. (3)

R gas constant, ft/$^{\circ}$R

Re Reynolds number

r_h hydraulic radius, ft ($r_h = D/4$ for round tubes)

s entropy, ft-lb/(lb)($^{\circ}$R)

T temperature, $^{\circ}$R

u velocity, ft/sec

v specific volume, cu ft/lb

w weight flow rate, lb/sec

x distance, ft

γ ratio of specific heats, c_p/c_v

μ viscosity, lb/(sec)(ft)

ρ density, lb/cu ft

τ temperature ratio, T_2/T_1

φ,ψ correlating parameters, eq. (15)

Subscripts:

b evaluated at bulk conditions

f evaluated at film conditions

h surface coefficient, Btu/(sec)($^{\circ}$R)

w wall

1 inlet

2 outlet

0 initial constant

APPENDIX B

ONE-DIMENSIONAL FLOW RELATIONS

For steady flow $(\partial/\partial t = 0)$ the relations governing continuity of mass, momentum, and an energy relation from equations (1), (2), and (5) are

$$\frac{d}{dx} \rho A u = 0 \qquad \text{(Mass)}$$

$$\frac{dp}{\rho} + \frac{u\,du}{g} + \frac{f}{r_h} \frac{u^2}{2g} dx = 0 \qquad \text{(Momentum)}$$

$$\frac{q}{u} + \frac{f}{r_h} \frac{u^2}{2g} dx = c_p\, dT - v\, dp \qquad \text{(Energy)}$$

For flow passages of constant cross-sectional area, the continuity of mass is expressed as

$$\frac{d\rho}{\rho} + \frac{du}{u} = 0$$

The equation of state for a perfect gas in differential form is

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

Substituting into the momentum and energy equations (eqs. (2) to (4)) yields

$$\frac{dp}{\rho} - \frac{u^2}{g} \frac{dp}{p} + \frac{u^2}{g} \frac{dT}{T} + \frac{f}{2r_h} \frac{u^2}{g} dx = 0$$

$$\left(\frac{Q}{\rho u r_h} + \frac{f}{2r_h} \frac{u^2}{g}\right) dx = c_p\, dT - \frac{dp}{\rho}$$

Eliminating dT, substituting $p = \rho RT$, and collecting terms lead to

$$\left(1 - \frac{u^2}{gRT} + \frac{u^2}{gTc_p}\right) \frac{dp}{\rho} + \frac{u^2}{gTc_p} \frac{Q\,dx}{\rho u r_h} + \left(1 + \frac{u^2}{gTc_p}\right) \frac{f}{2r_h} \frac{u^2}{g} dx = 0 \qquad \text{(B1)}$$

The following relations are based on the sonic velocity $C^2 = \gamma gRT$ and the Mach number $M^2 = u^2/C^2$:

$$\frac{u^2}{gRT} = \gamma M^2$$

$$\frac{u^2}{gTc_p} = \frac{u^2}{\gamma gRT} \frac{\gamma R}{c_p} = (\gamma - 1) M^2$$

Assume now that the Mach number is small relative to 1. Consequently, the static-temperature rise is related to the heat input by

$$wc_p \, dT = Q \frac{A \, dx}{r_h} \tag{B2}$$

After second-order Mach number terms are dropped and the expression is multiplied through by $p\rho = \rho \cdot \rho RT$, these substitutions yield

$$p \, dp + \frac{R}{g}\left(\frac{w}{A}\right)^2 \left(dT + \frac{f}{2r_h} T \, dx\right) = 0 \tag{B3}$$

Integrating along the flow passage gives

$$\frac{p_2^2 - p_1^2}{2} + \frac{R}{g}\left(\frac{w}{A}\right)^2 \left(T_2 - T_1 + \frac{1}{2r_h} \int_{x=0}^{x} fT \, dx\right) = 0 \tag{B4}$$

The friction factor f is approximated as a function of Reynolds number, or

$$f(x) = \frac{f_0}{Re^n} = \frac{f_0 \mu^n}{\left[4r_h\left(\frac{w}{A}\right)\right]^n} \tag{B5}$$

and the viscosity μ is assumed a function of temperature:

$$\mu(T) = \mu_0 T^m \tag{B6}$$

Linearizing the pressure-difference term and substituting the friction factor and viscosity assumptions (eqs. (B5) and (B6)) into equation (B4) lead to

$$p \, \Delta p = \frac{R}{g}\left(\frac{w}{A}\right)^2 \left\{T_2 - T_1 + \frac{1}{2r_h} \frac{f_0 \mu_0^n}{\left[4r_h\left(\frac{w}{A}\right)\right]^n} \int_{x=0}^{x} T^{mn+1} \, dx\right\} \tag{B7}$$

Finally, assume a constant heat-input distribution along the tube, so that $Q(x) = Q_0$. Then, from equation (B2),

$$dT = \frac{Q_0}{r_h\left(\frac{w}{A}\right)c_p} dx$$

$$T = T_1 + \frac{Q_0 x}{r_h\left(\frac{w}{A}\right)c_p} \tag{B7a}$$

The integral in equation (B7) is evaluated as

$$\int_{x=0}^{x} T^{mn+1}\,dx = \int_{x=0}^{x}\left[T_1 + \frac{Q_0 x}{r_h\left(\frac{w}{A}\right)c_p}\right]^{mn+1} dx$$

$$= \frac{1}{(mn + 2)\left[\frac{Q_0}{r_h\left(\frac{w}{A}\right)c_p}\right]}\left\{\left[T_1 + \frac{Q_0 x}{r_h\left(\frac{w}{A}\right)c_p}\right]^{mn+2}\right\}_{x=0}^{x}$$

$$= \frac{1}{(mn + 2)\left(\frac{T_2 - T_1}{x}\right)}\left(T_2^{mn+2} - T_1^{mn+2}\right)$$

$$= \frac{T_1^{mn+1}x}{(mn + 2)}\,\frac{\left(\tau^{mn+2} - 1\right)}{\tau - 1}$$

where the temperature ratio $\tau \equiv T_2/T_1$. Substituting back into equation (B7) and simplifying by virtue of equations (B5) and (B6) yield

$$p\,\Delta p = \frac{R}{g}\left(\frac{w}{A}\right)^2\left[T_2 - T_1 + \frac{f_1 x}{2r_h}\,\frac{T_1}{(mn + 2)}\,\frac{\tau^{mn+2} - 1}{\tau - 1}\right]$$

or, finally

$$p\,\Delta p = \frac{RT}{g}\left(\frac{w}{A}\right)^2\left[\tau - 1 + \frac{f_1 x}{2r_h(mn + 2)}\,\frac{\tau^{mn+2} - 1}{\tau - 1}\right] \tag{B8}$$

It is convenient, however, to use the pressure drop in ratio form. In that case, with Mach number as the coefficient, equation (B8) becomes

$$\frac{\Delta p}{p} = \gamma M^2\left(\tau - 1 + \frac{f_1 x}{2r_h}\,\frac{1}{mn + 2}\,\frac{\tau^{mn+2} - 1}{\tau - 1}\right) \tag{B9}$$

The possible minimum value of pressure drop $\Delta p/p$ as weight flow is varied at constant heat input is determined from equation (B8). First, the equation is written with weight flow per unit area as the primary variable:

$$p\,\Delta p = \frac{RT}{g}\left(\frac{w}{A}\right)^2\left(\frac{Q_0 x}{r_h\left(\frac{w}{A}\right)c_pT_1} + \frac{\dfrac{f_0\mu_1^n}{\left(4r_h\,\frac{w}{A}\right)^n}\,\dfrac{x}{2r_h(mn + 2)}}{\dfrac{Q_0 x}{r_h\left(\frac{w}{A}\right)c_pT_1}}\left\{\left[1 + \frac{Q_0 x}{r_h\left(\frac{w}{A}\right)c_pT_1}\right]^{mn+2} - 1\right\}\right)$$

$$\left(\frac{\partial p\,\Delta p}{\partial\,\frac{w}{A}}\right)_{Q_0} = \frac{RT}{g}\left[\frac{Q_0 x}{r_hc_pT_1} + \frac{\dfrac{f_0\mu_1^n}{\left(4r_h\,\frac{w}{A}\right)^n}\,\dfrac{x}{2r_h}}{\dfrac{Q_0}{r_h\left(\frac{w}{A}\right)c_pT_1}}\left(\frac{w}{A}\right)\left(\frac{3 - n}{mn + 2}\left\{\left[1 + \frac{Q_0 x}{r_h\left(\frac{w}{A}\right)c_pT_1}\right]^{mn+2} - 1\right\} - \frac{Q_0 x}{r_h\left(\frac{w}{A}\right)c_pT_1}\left[1 + \frac{Q_0 x}{r_h\left(\frac{w}{A}\right)c_pT_1}\right]^{mn+1}\right)\right] \tag{B10}$$

$$\left(\frac{\partial p\,\Delta p}{\partial\,\frac{w}{A}}\right)_{Q_0} = \frac{RT}{g}\left\{\frac{Q_0 x}{r_hc_pT_1} + \frac{f_1 x}{2r_h}\,\frac{w}{A}\left[\frac{3 - n}{mn + 2}\,\frac{\tau^{mn+2} - 1}{\tau - 1} - \tau^{mn+1}\right]\right\} \tag{B11}$$

The first term represents the momentum pressure drop and is always positive if the heat is added to the fluid. Thus, for the slope to be negative the bracketed term must be negative, or

$$\frac{3 - n}{mn + 2}\,\frac{\tau^{mn+2} - 1}{\tau - 1} - \tau^{mn+1} \leq 0 \tag{B12}$$

For large temperature ratios, where $\tau^{mn+2} \gg 1$,

$$(3 - n)\tau - (mn + 2)(\tau - 1) \leq 0$$

and for a negative slope to occur

$$n + mn > 1$$

This relation is discussed in the ANALYSIS section of the report.

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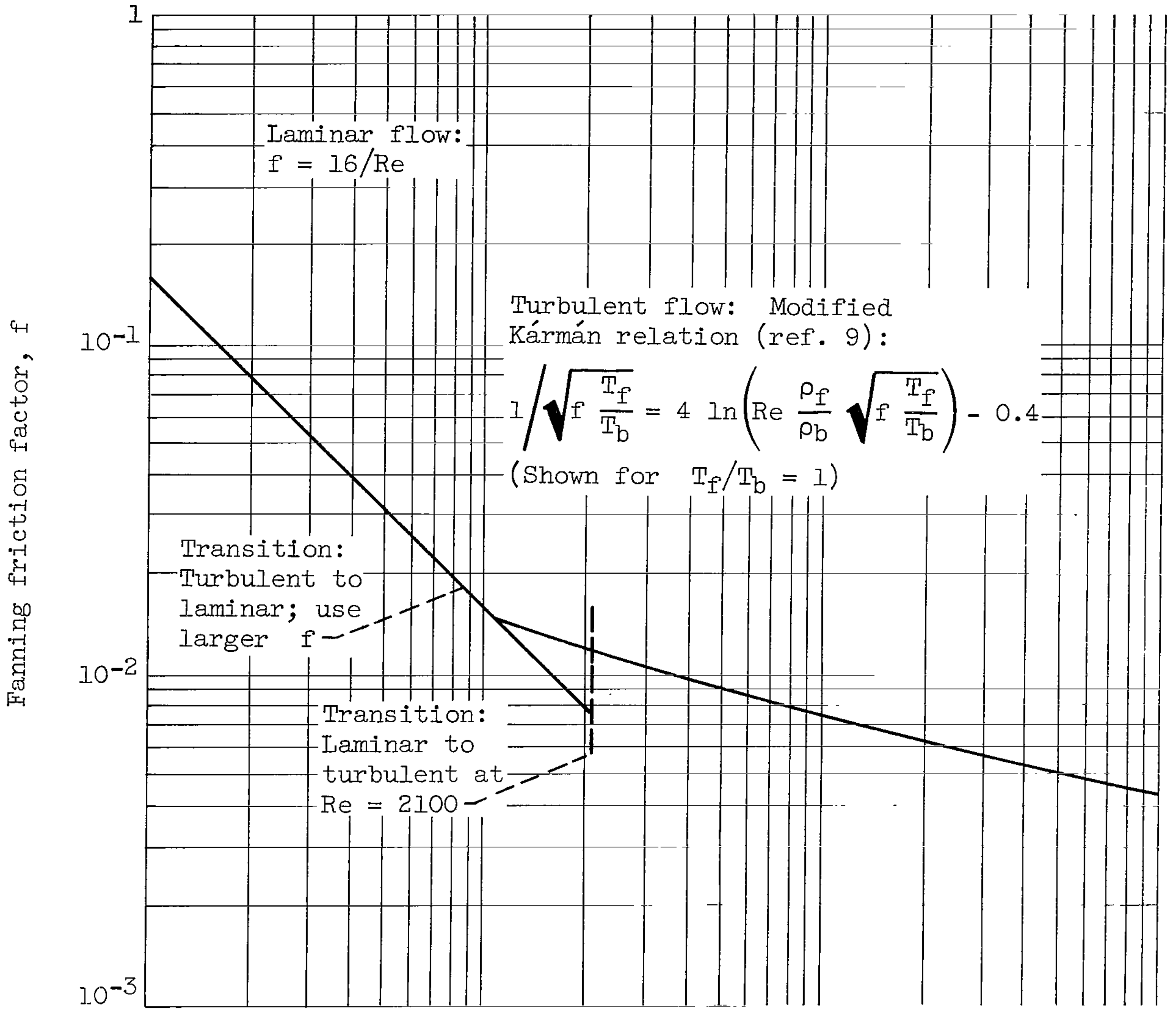
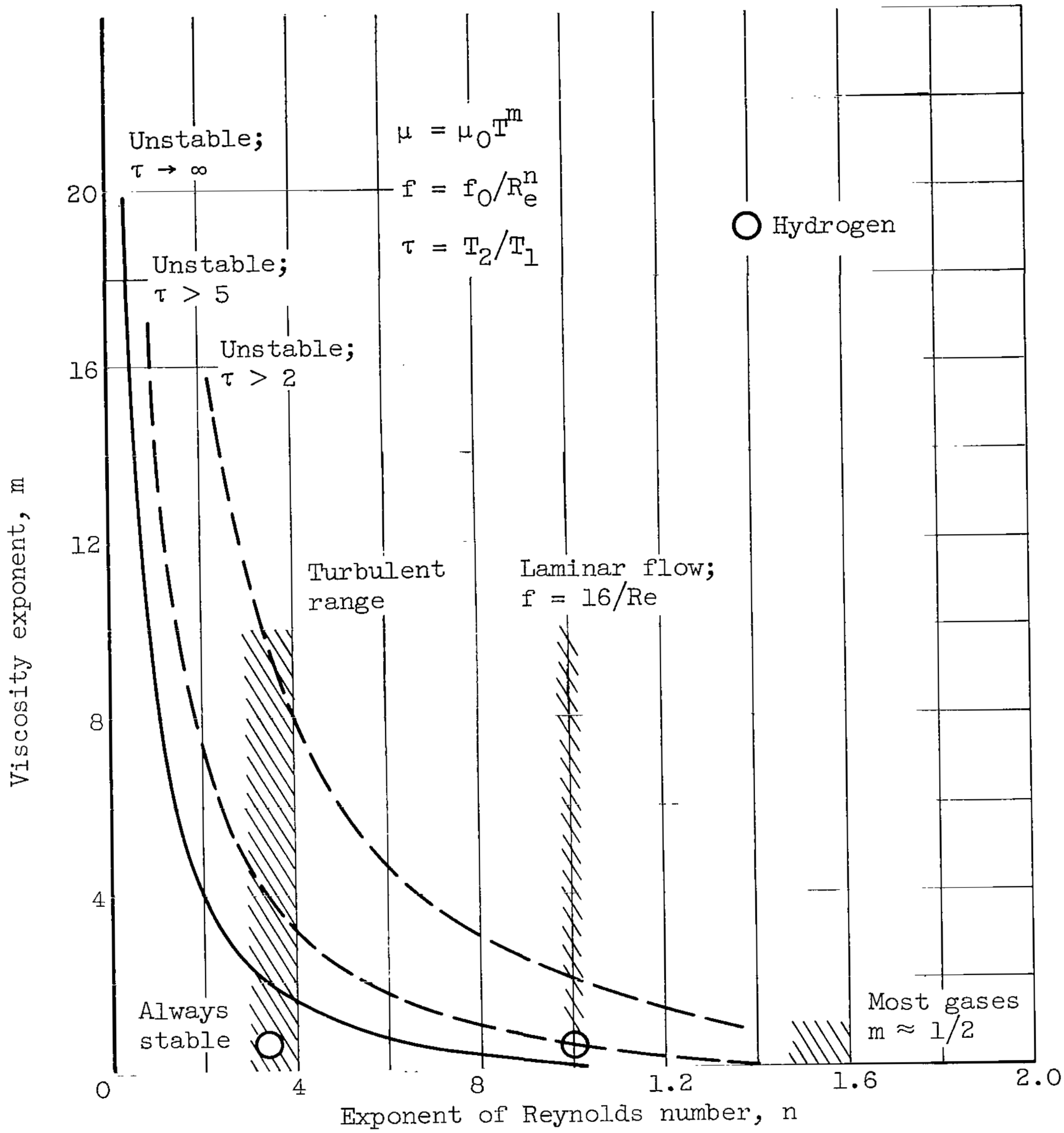
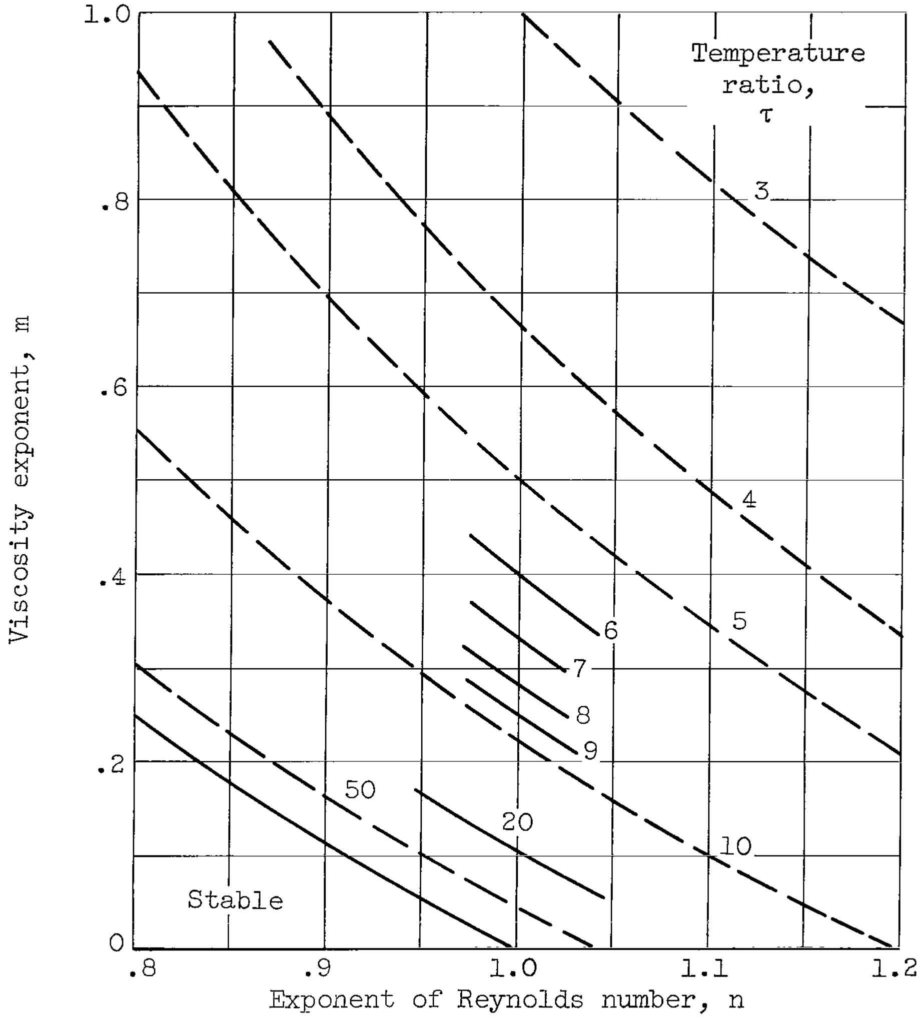


Figure 1. - Assumed laminar-turbulent flow transitions.



(a) General trends.

Figure 2. - Stability criteria from approximate solution; $Q(x) = Q_0$.



(b) Detail of laminar flow region.

Figure 2. - Concluded. Stability criteria from approximate solution; $Q(x) = Q_D$.

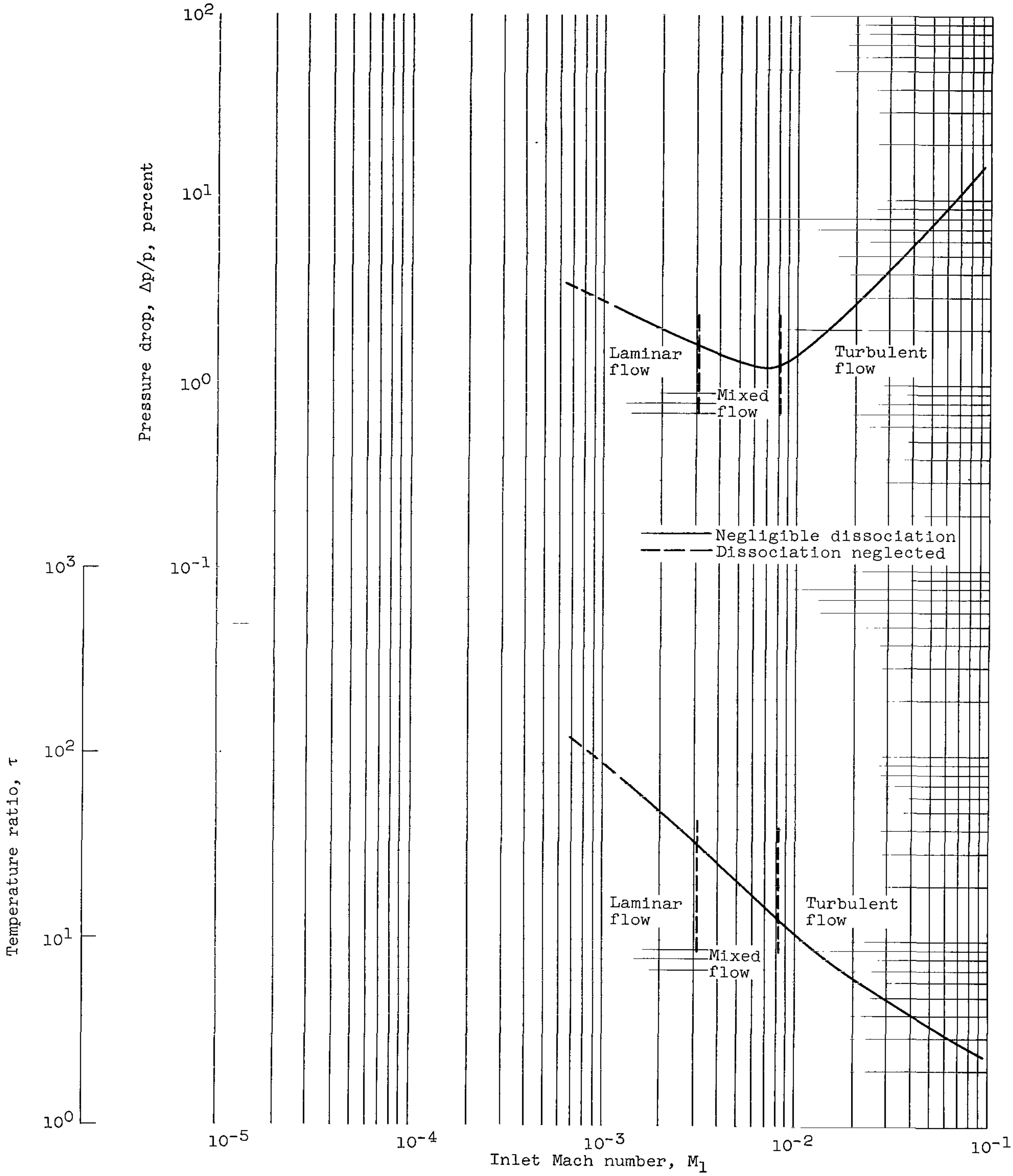


Figure 3. - A typical instability characteristic presented on logarithmic coordinates. Inlet pressure, 20 pounds per square inch absolute; inlet temperature, 50° R; tube diameter, 0.10 inch; tube length, 4 feet; heat input, 1 Btu per second per square foot.

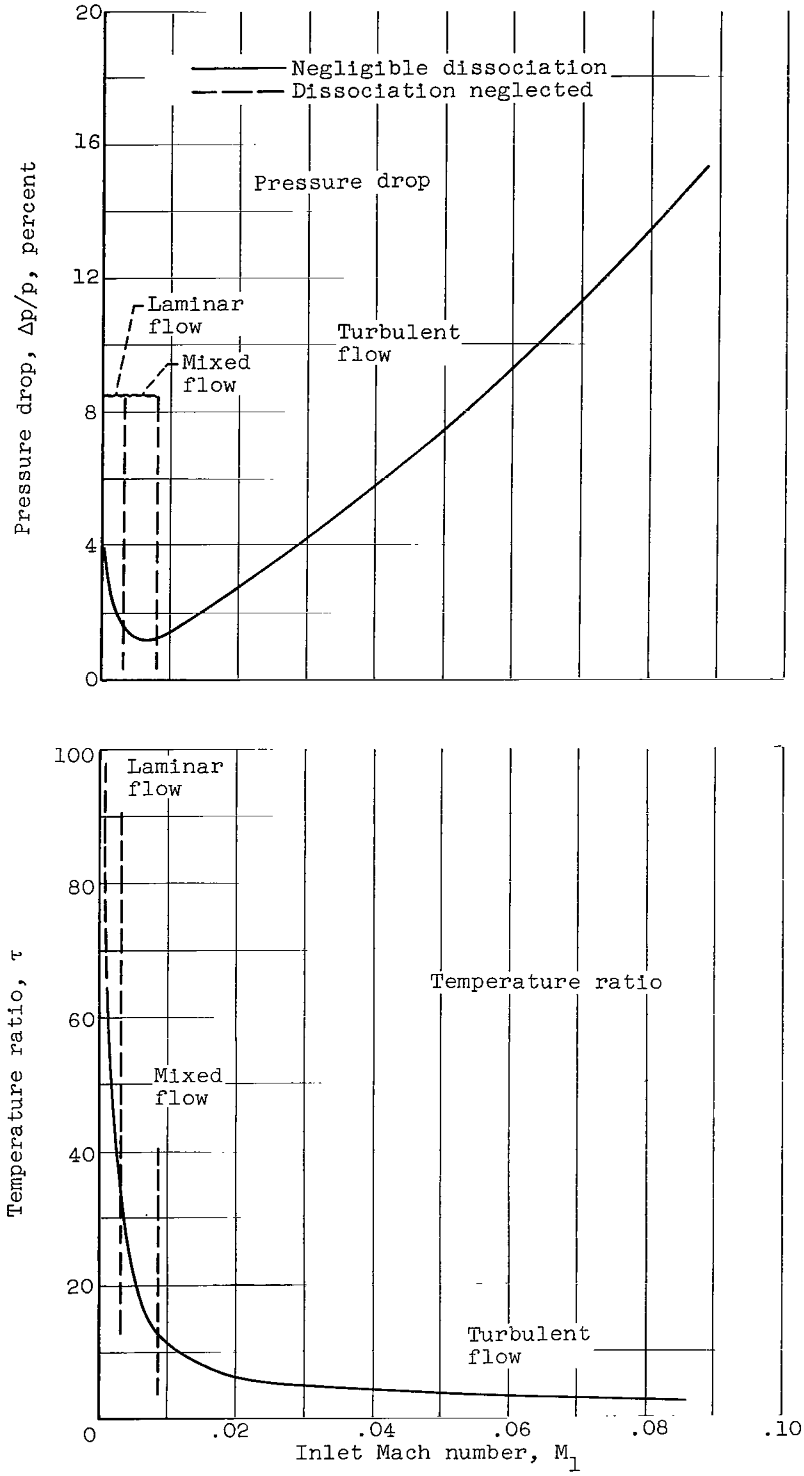


Figure 4. - A typical instability characteristic presented on linear coordinates. Inlet pressure, 20 pounds per square inch absolute; tube diameter, 0.10 inch, tube length, 4 feet; heat input, 1 Btu per second per square foot.

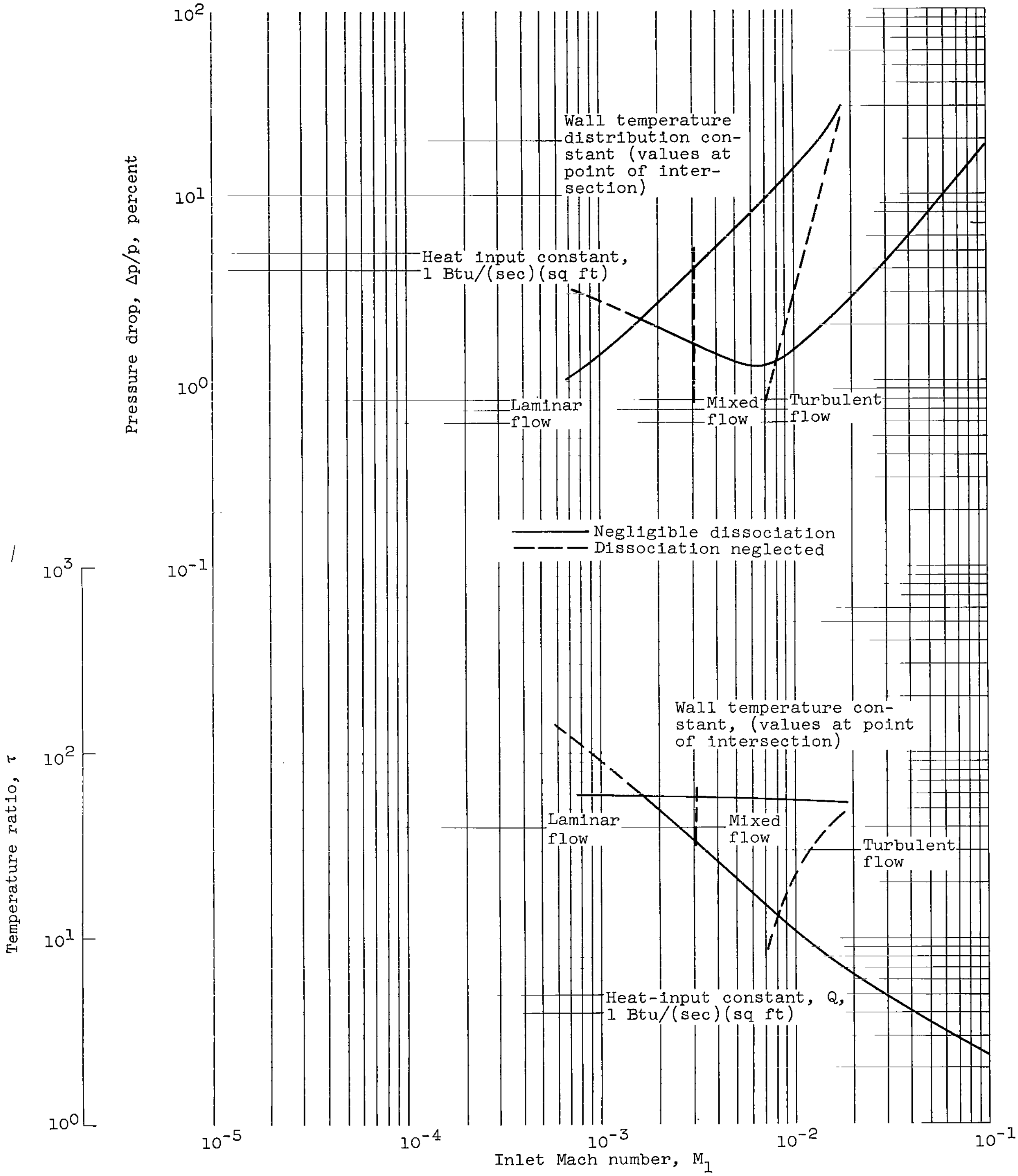
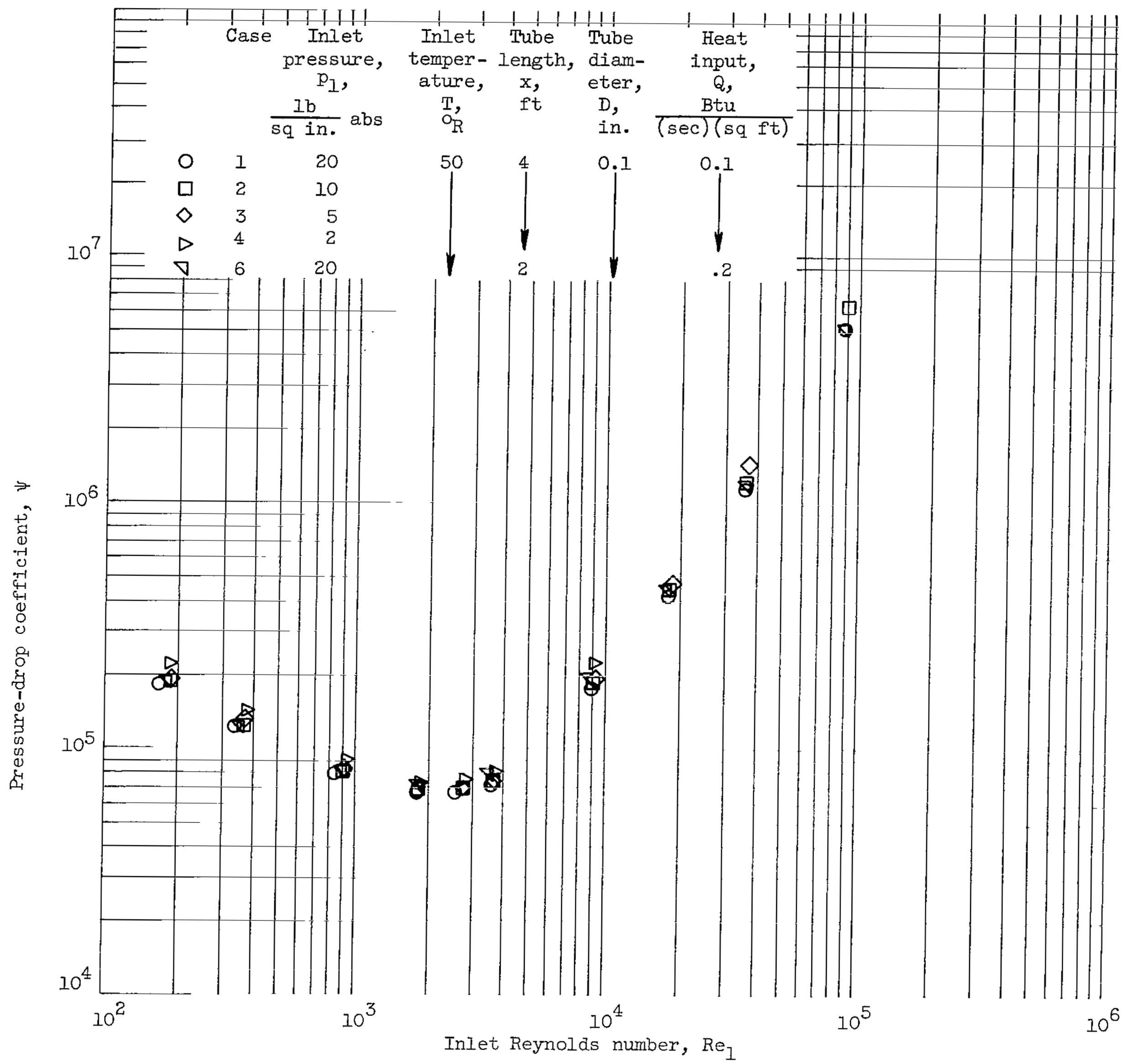
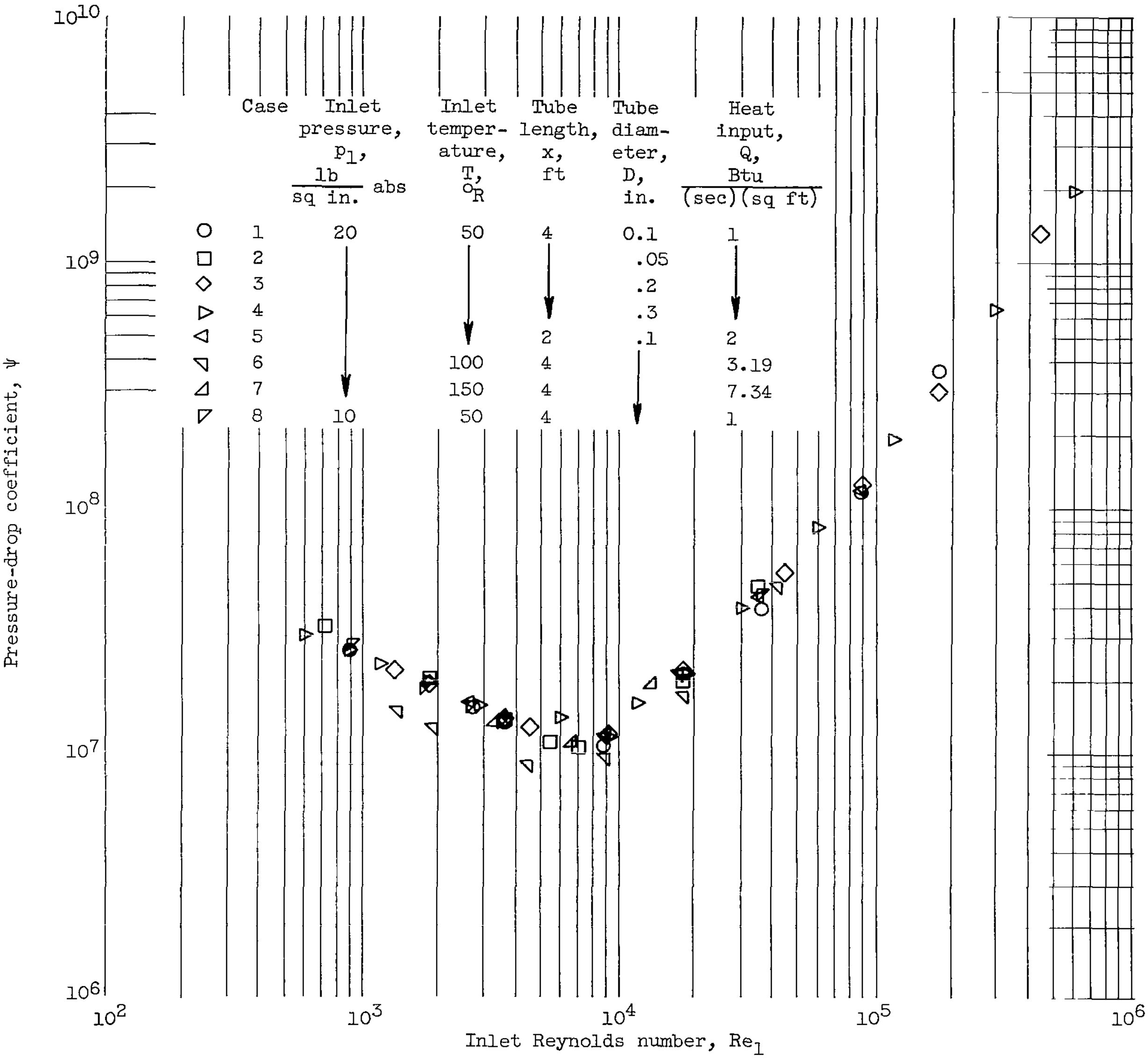


Figure 5. - Comparison of constant heat input and constant wall temperature trends. Inlet pressure, 20 pounds per square inch absolute; inlet temperature, 50° R; tube diameter, 0.10 inch; tube length, 4 feet.



(a) Heat-input coefficient, φ, 3400 ±2 percent.

Figure 6. - Evaluation of correlating parameters.



(b) Heat-input coefficient, φ, 34,000 ±2 percent.

Figure 6. - Concluded. Evaluation of correlating parameters.

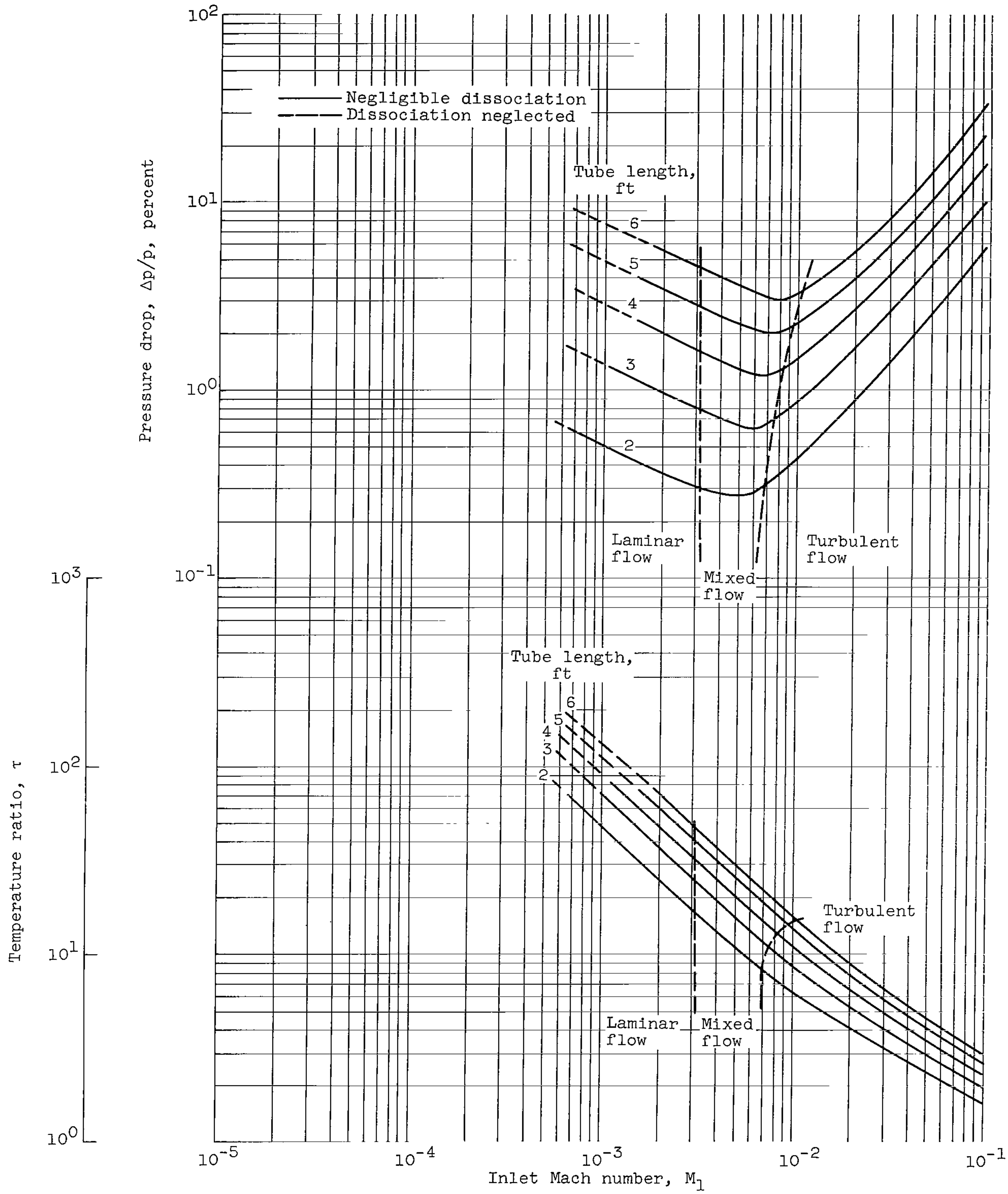


Figure 7. - Effect of tube length on stability criteria. Inlet pressure, 20 pounds per square inch absolute; inlet temperature, 50° R; tube diameter, 0.10; heat input, 1 Btu per second per square foot.

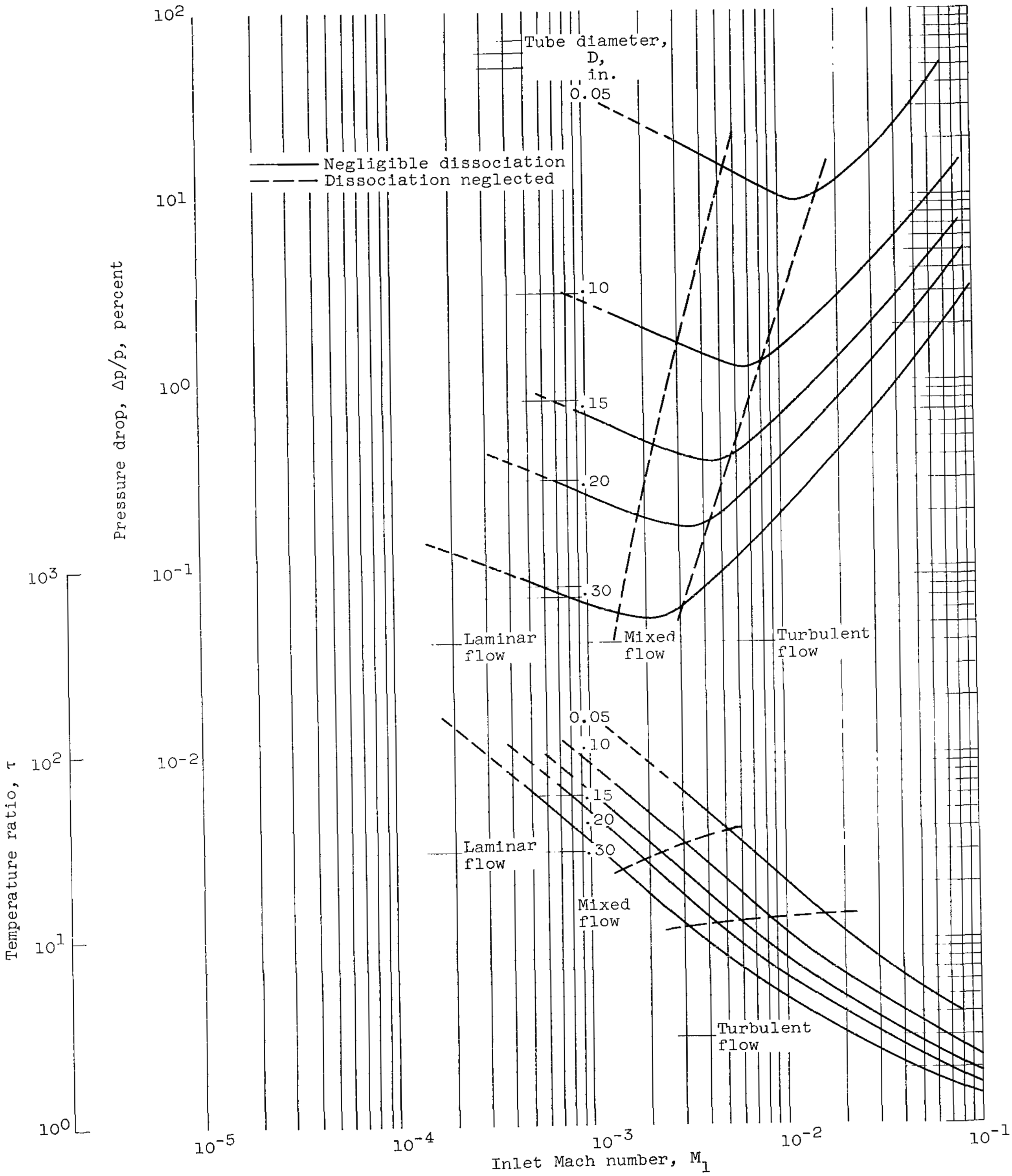
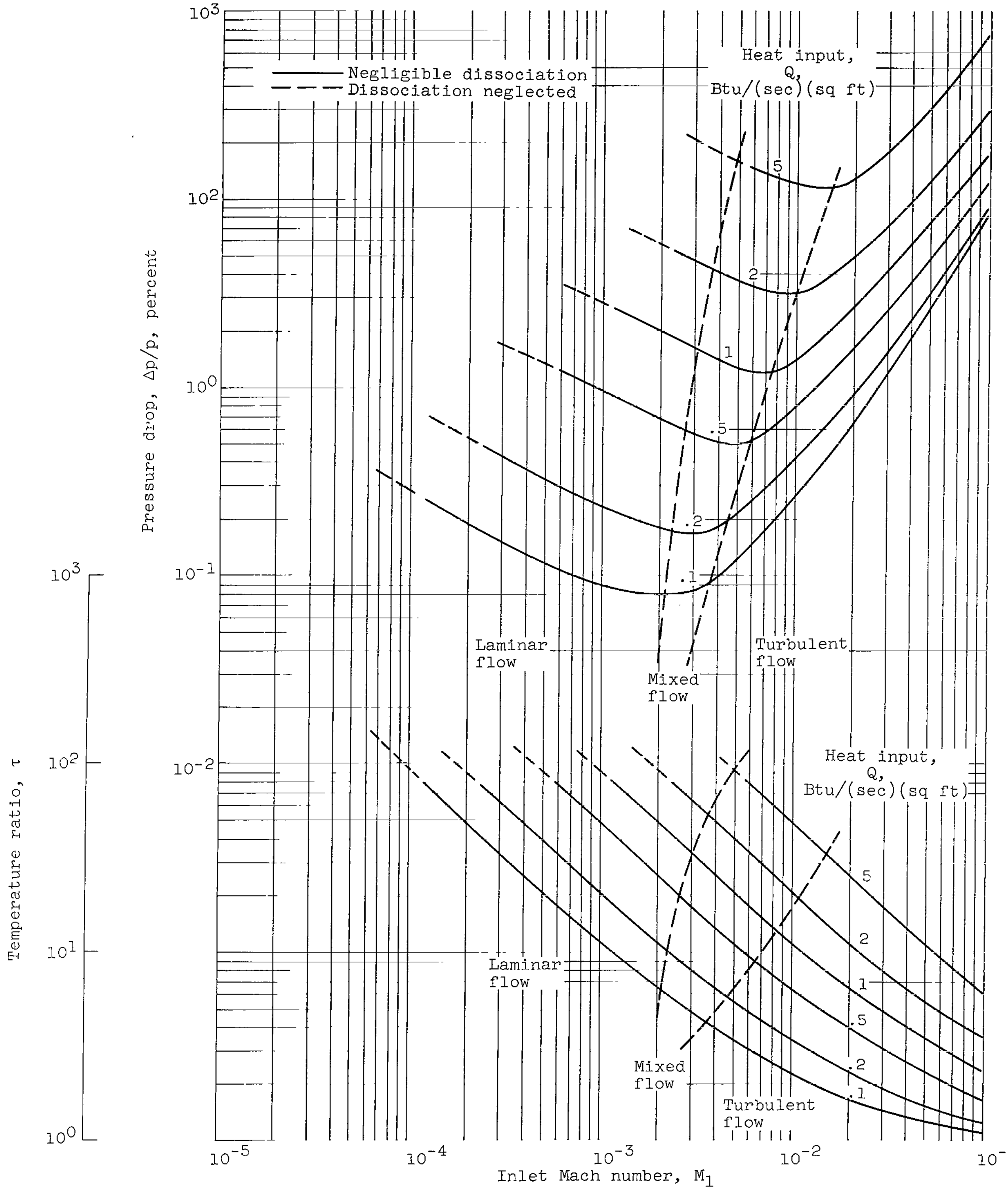
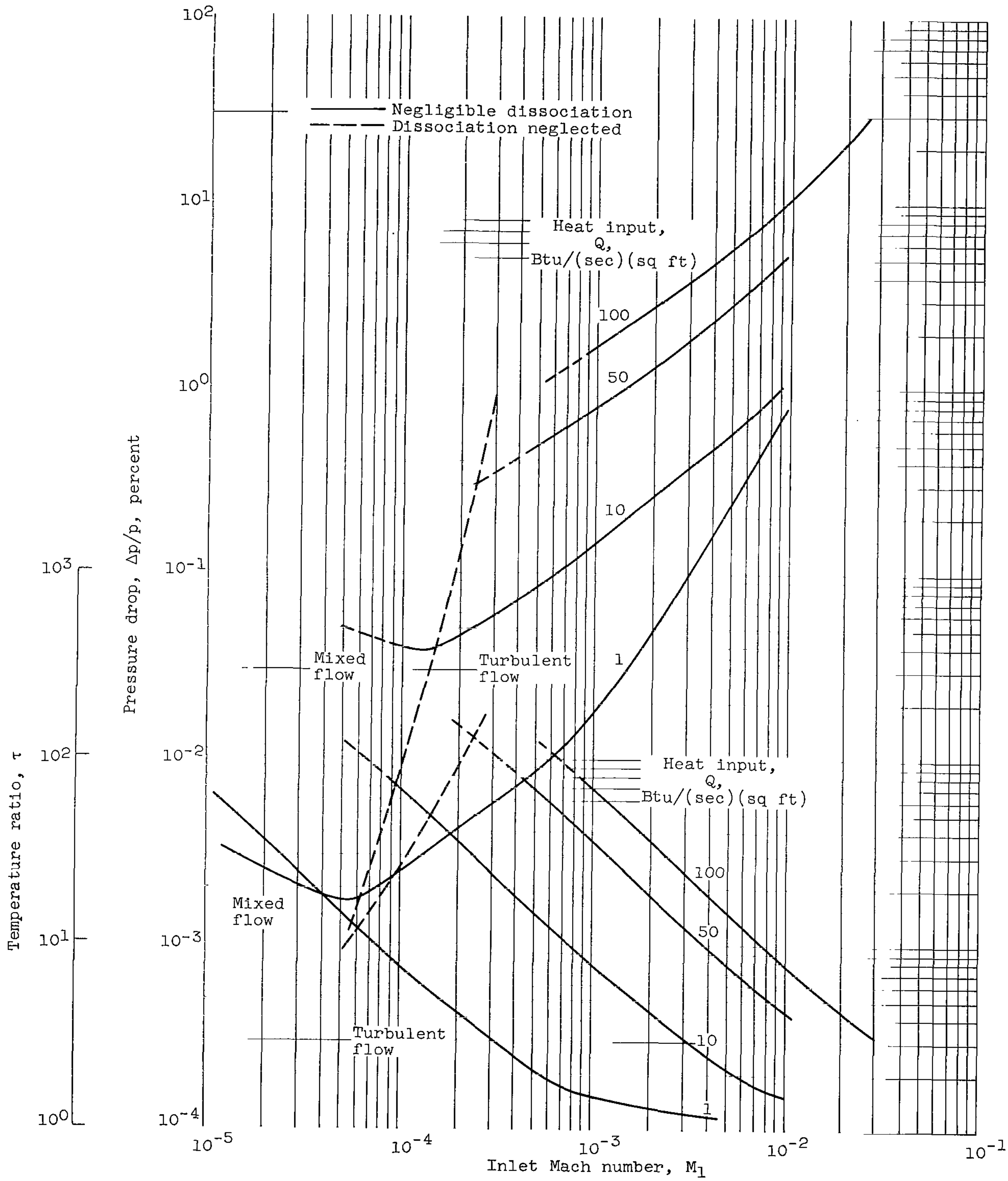


Figure 8. - Effect of tube diameter on stability criteria. Inlet pressure, 20 pounds per square inch absolute; inlet temperature, 50° R; tube length, 4 feet; heat input, 1 Btu per second per square foot.



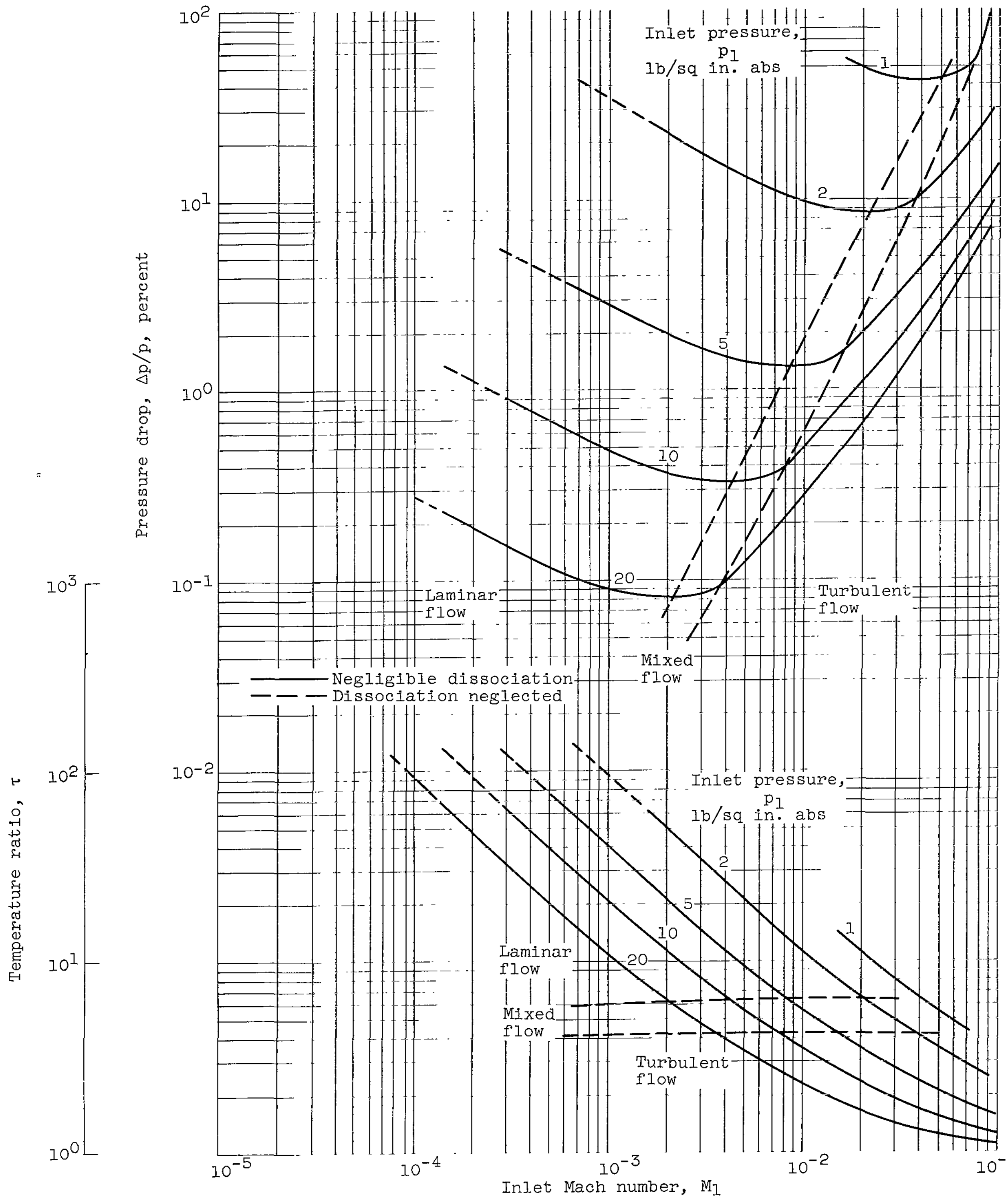
(a) Inlet pressure, 20 pounds per square inch absolute.

Figure 9. - Effect of heat input on stability criteria. Inlet temperature, 50° R; tube diameter, 0.10 inch; tube length, 4 feet.



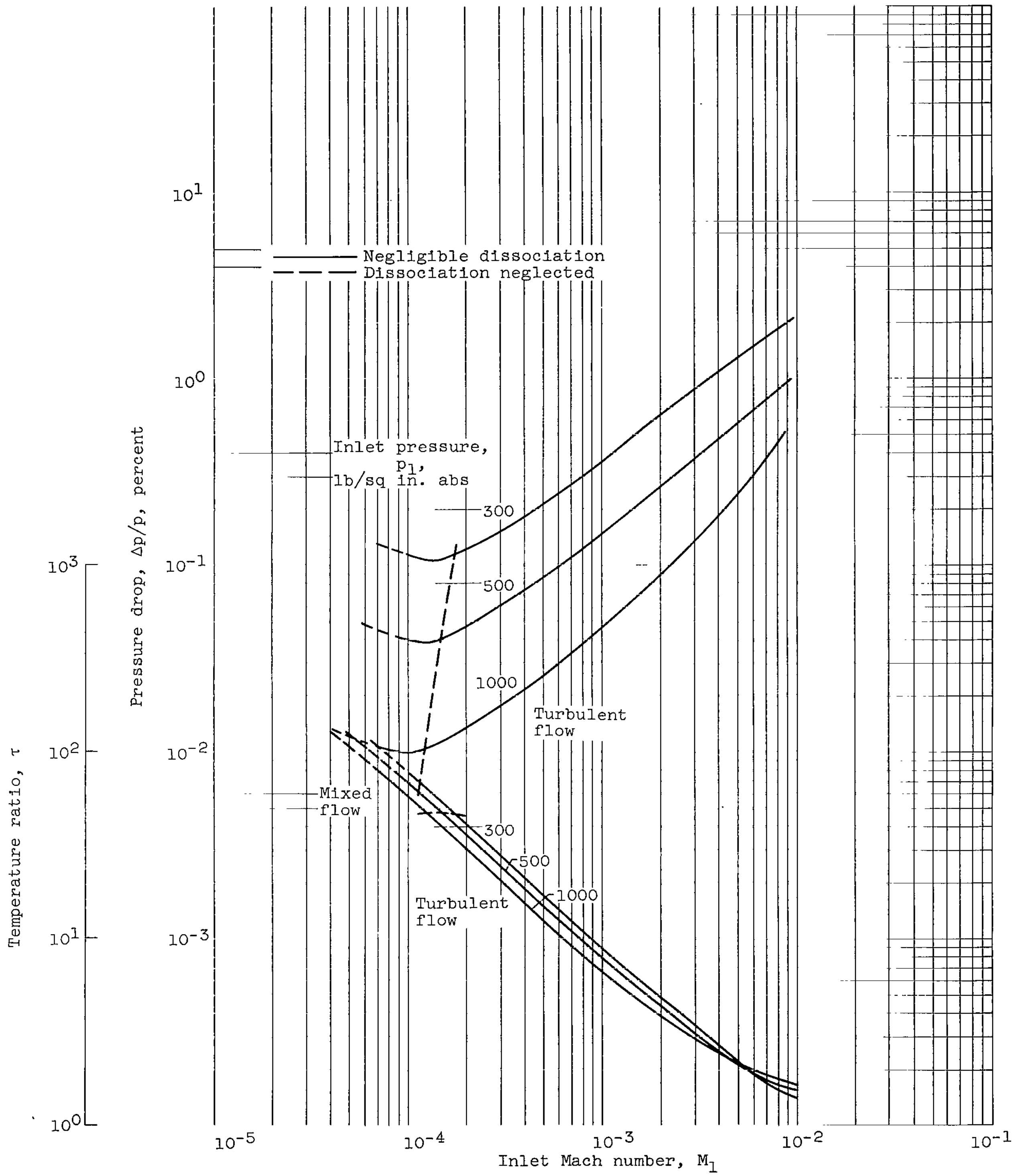
(b) Inlet pressure, 500 pounds per square inch absolute.

Figure 9. - Concluded. Effect of heat input on stability criteria. Inlet temperature, 50° R; tube diameter, 0.10 inch; tube length, 4 feet.



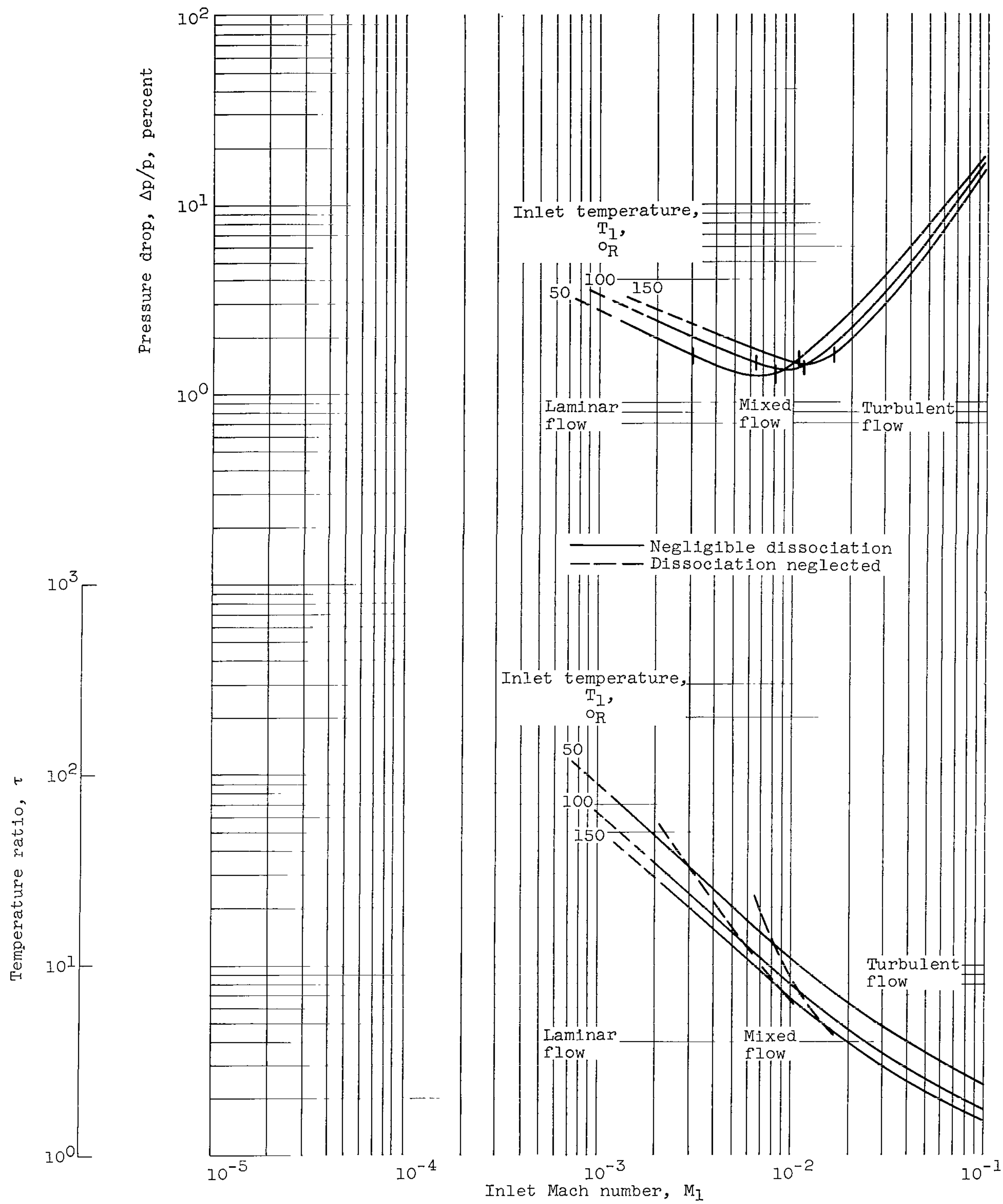
(a) Low pressure range with heat input of 0.1 Btu per second per square foot.

Figure 10. - Effect of pressure level on stability criteria. Inlet temperature, 50° R, tube diameter, 0.10 inch; tube length, 4 feet.



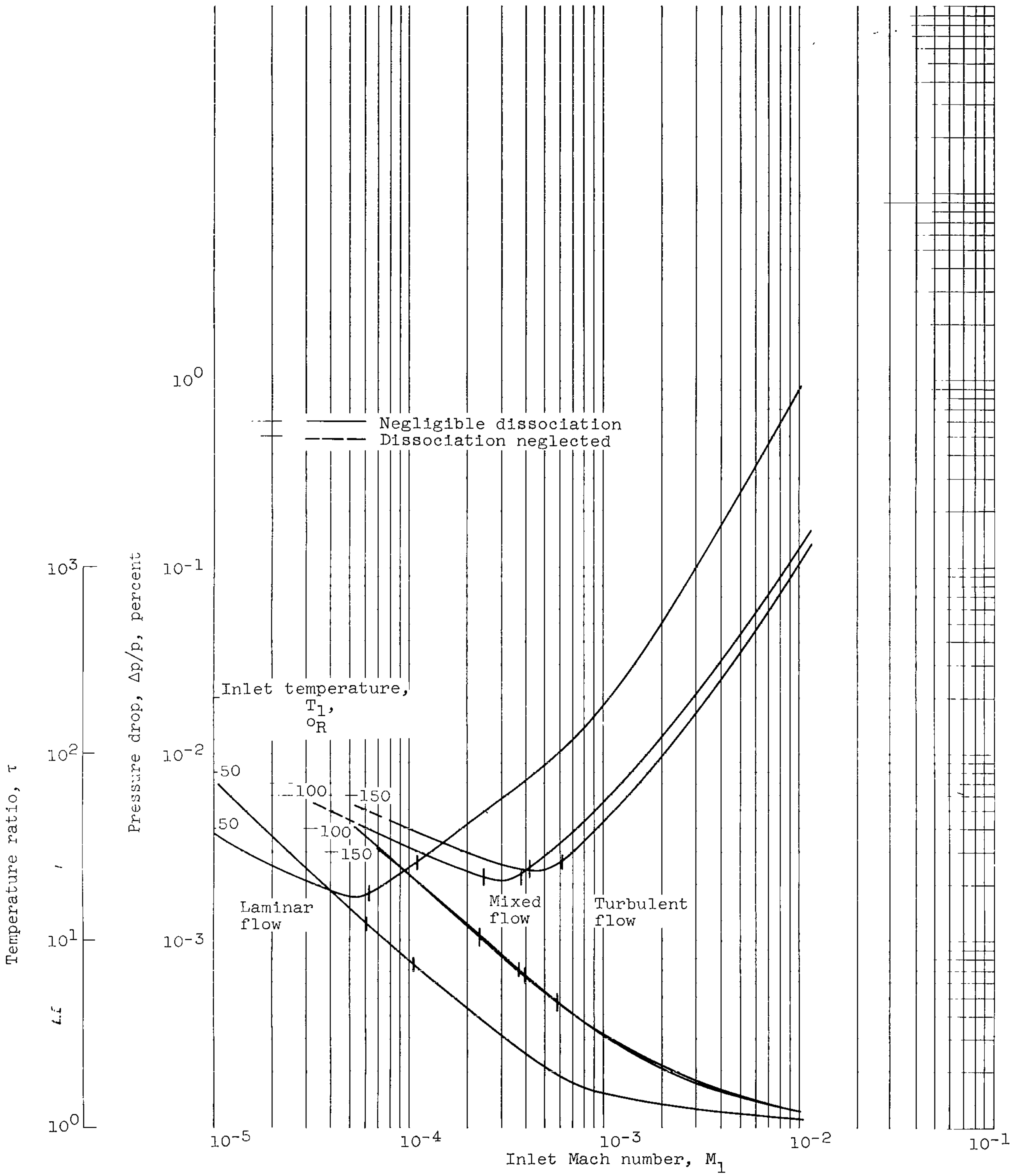
(b) High pressure range with heat input of 10 Btu per second per square foot.

Figure 10. - Concluded. Effect of pressure level on stability criteria. Inlet temperature, 50° R; tube diameter, 0.10 inch; tube length, 4 feet.



(a) Low pressure range with inlet pressure of 20 pounds per square absolute and heat input of 1 Btu per second per square foot.

Figure 11. - Effect of inlet temperature on stability criteria. Tube diameter, 0.10 inch; tube length, 4 feet.



(b) High pressure range with inlet pressure of 500 pounds per square inch absolute and heat input of 1 Btu per second per square foot.

Figure 11. - Concluded. Effect of inlet temperature on stability criteria. Tube diameter, 0.10 inch; tube length, 4 feet.